

# Precise Semantics for Uncertainty Modeling (PSUM)

Version 1.0

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# Contents

Pr	Preface					
1	Scope					
2	Conformance					
3	References	5				
	3.1 Normative References	5				
	3.2 Non-normative References	5				
4	Terms and Definitions					
5	Symbols					
6	Additional Information					
	6.1 Guide to this Specification	11				
	6.1.1 Document Conventions	11				
	6.1.2 Clause Structure	11				
	6.2 Intellectual Property Rights	11				
	6.3 Changes to Existing OMG Specifications					
	6.4 Acknowledgments	11				
7	Background and Rationale					
8	PSUM Overview					
9	Overall					
10	0 Belief					
11	1 Uncertainty					
12	2 Measurement					

### 13 Evidence

A	Exai	amples of Applying PSUM				
	A.1	1 Specifying Uncertainty Requirements				
	A.2	2 Measurement Uncertainties				
		A.2.1	Indicating the Precision of a Measurement	32		
		A.2.2	Adding Confidence to a Belief	34		
		A.2.3	Working with two or more Belief Statements	35		
		A.2.4	Assigning Uncertainty to Uncertainty	36		
	A.3	Operat	ional Uncertainty due to Inconsistent Information	37		
Bi	Sibliography					

#### 29

# Preface

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Times/Times New Roman - 10 pt.: Standard body text

Helvetica/Arial - 10 pt. Bold: OMG Interface Definition Language (OMG IDL) and syntax elements.

Courier - 10 pt. Bold: Programming language elements.

Helvetica/Arial - 10 pt: Exceptions

**NOTE:** Terms that appear in italics are defined in the glossary. Italic text also represents the name of a document, specification, or other publication.

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# 1 Scope

The purpose of this standard is to specify the Precise Semantics for Uncertainty Modeling (PSUM) version 1.0 to build the foundation for developing uncertainty modeling solutions, to guide the implementation of uncertainty modeling tools, to provide the basis for developing training materials and resources in the application of uncertainty modeling, and to serve as the cornerstone of proposing future revisions and extensions of uncertainty modeling solutions.

PSUM v1.0 is an uncertainty modeling metamodel, which 1) captures uncertainty and its related concepts, and 2) enables measurements of uncertainty and uncertainty-related concepts. PSUM v1.0 is intentionally designed to be generic such that it acts as the foundation for developing other modeling languages, e.g., integrating with UML or SysML for achieving specific modeling purposes and user applications, and being specialized for developing domain-specific uncertainty modeling solutions.

The PSUM metamodel will be defined using the MOF meta-modeling language, to be consistent with other models defined by OMG. Therefore, it will have a standard textual representation in XMI, which can be used by software to check its conformance to the PSUM specification.

# 2 Conformance

PSUM defines a metamodel for defining and representing uncertainty and uncertainty related concepts, associating to SMM for quantifying uncertainty and other measurable concepts such as belief. To be PSUM compliant, an implementation must provide: 1) the capability to generate XMI documents based on the PSUM XMI schema from an existing model of the tool; and 2) the capability to import a model that is based on the PSUM XMI schema into the tool.

This specification defines two levels of conformance:

- Level 1 requires full implementation of PSUM except for the *Belief* package. In other words, one can start directly from the *Uncertainty* package to specify, characterize and measure uncertainties without specifying *Belief* and *BeliefStatement*.
- *Level 2* requires full implementation of PSUM including the *Belief* package, implying that an implementation needs to cover the complete PSUM metamodel.

Implementations of both levels need to include SMM, because PSUM refers to SMM for quantifying *Uncertainty* and other measurable elements. Quantification is an important aspect of uncertainty modeling.

Implementations of both levels can optionally include SACM. This is because PSUM defines *Evidence* and classifies it into four types (*EvidenceType*), which might be sufficient in some uncertainty modeling contexts.

# 3 References

## 3.1 Normative References

The following normative documents contain provisions which, through reference in this text, constitute provisions of this specification. For dated references, subsequent amendments to, or revisions of, any of these publications do not apply.

- [SACM] Structured Assurance Case Metamodel (SACM), version 2.3, OMG Specification https://www.omg.org/spec/SACM/2.3
- [SMM] Structured Metrics Metamodel (SMM), version 1.2, OMG Specification https://www.omg.org/spec/SMM/1.2
- [SysML] OMG Systems Engineering Modeling Language (SysML), version 1.6, OMG Specification https://www.omg.org/spec/SysML/1.6
  - [UML] Unified Modeling Language (UML), version 2.5.1, OMG Specification https://www.omg.org/spec/UML/2.5.1
  - [XMI] XML Metadata Interchange (XMI), version 2.5.1, OMG Specification https://www.omg.org/spec/XMI/2.5.1
- [ISO 31000] Risk management, International Standard https://www.iso.org/iso-31000-risk-management.html

## 3.2 Non-normative References

None.

# 4 Terms and Definitions

None.

# 5 Symbols

None.

# 6 Additional Information

(informative)

## 6.1 Guide to this Specification

### 6.1.1 Document Conventions

All clauses of this document are *normative* unless explicitly marked "(informative)". The marking "(informative)" of a particular clause also applies to all contained sub-clauses of that clause.

Specification text is written in normal Roman font. Model elements are referenced within the text using *Italic font*. OCL constraints are written in Code font.

### 6.1.2 Clause Structure

Clause 7 is an informative overview of the principles of uncertainty. Clause 8 defines the abstract architecture of the PSUM metamodel. Clause 9 introduces the foundational elements of the PSUM metamodel. Clause 10 defines the Belief package. Clause 11 defines the Uncertainty concepts of PSUM. Clause 12 define the Measurement model used by PSUM. Clause 13 defines the PSUM Evidence model as derived from SACM. Annex A provides examples how to use PSUM for uncertainty modeling.

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## 6.4 Acknowledgments

The following organizations submitted this specification:

- 88solutions Corporation
- Simula Research Laboratory
- SOFTEAM (Docaposte Group)

The following organizations contributed to this specification:

- Thematix Partners LLC
- University of Duisburg-Essen
- University of Málaga
- Helmut Schmidt University

# 7 Background and Rationale

### (informative)

Uncertainty has been studied in various fields, such as philosophy, physics, statistics, and finance, to describe a situation of lacking knowledge about the state of a system and/or potential future outcome(s) [1]. In the literature, various efforts (e.g., [2, 3, 4, 5]) have been made to understand and classify uncertainty for various purposes (e.g., supporting decision-making), from different perspectives (e.g., ethics), and in diverse domains (e.g., healthcare).

As defined in [6], *uncertainty* means "the lack of confidence (i.e., knowledge) about the timing and nature of inputs, the state of a system, a future outcome, as well as other relevant factors". In the ISO 31000 standard [1], *uncertainty* is defined in the context of Risk Management as "the state, even partial, of deficiency of information related to, understanding or knowledge of an event, its consequence, or likelihood."

It is therefore important to *face, understand*, and explicitly *specify/model* uncertainty in system/software engineering. "Facing" uncertainty means identifying uncertainties explicitly, analyze them, and make an effort to deal with them in all phases of development, including, for instance, evaluating effects of uncertainties, reasoning about how uncertainty propagates (with the goal to reduce it when possible), or at least constraining and alleviating its consequences. To "understand" uncertainty, it is necessary to clearly and precisely define it and all its relevant concepts. "Specifying" uncertainty means, wherever applicable, a precise formulation of uncertainty should be explicitly associated with various development artifacts, such as requirements, analysis, and design models, or test-ready models (to enable the generation of test cases for testing systems/software under uncertainties).

Uncertainty is gradually gaining more and more attention in system/software engineering these days. In particular, complex software-intensive systems, such as Cyber-Physical Systems (CPSs) and self-adaptive systems, typically operate in dynamic and unpredictable environments. A survey of how uncertainty has been specified so far in software models is presented in [7]. In [8], the authors compiled different categorizations of uncertainties in the context of self-adaptive systems. However, a unified and generally applicable conceptual fundamental and common terminology for specifying uncertainty is missing, which also includes a precise elaboration of relationships between uncertainty and other related concepts such as measurements. Specifically, potential applications of uncertainty modeling include: (1) specifying uncertainty requirements in use case models, (2) modeling uncertainties as part of SysML or UML analysis and design models, (3) modeling uncertainties as part of test-ready models for enabling model-based testing, which might be specified with extensions to SysML, UML and/or MARTE and UTP V2.0, (4) modeling uncertainties as part of BPMN models, and (5) modeling uncertainty could then be used as the basis for performing different analyses, such as discovering unanticipated uncertainties or generating other artifacts such as test cases.

This Precise Semantics for Uncertainty Modeling (PSUM) specification covers in particular the following two aspects: (1) Capturing uncertainty and its related concepts; (2) Enabling measurements of uncertainty and uncertainty-related concepts.

# 8 **PSUM Overview**

The PSUM metamodel is composed of three top packages: *Overall, Core, and Measurement.* The *Core* package is composed of the sub-packages of *Evidence* and *Risk, Uncertainty, and Belief.* In addition, the *Evidence* package of PSUM is dependent on the SACM specification [9], while the *Measurement* package of PSUM is dependent on the SMM specification [10], as shown in Figure 8.1 given below.

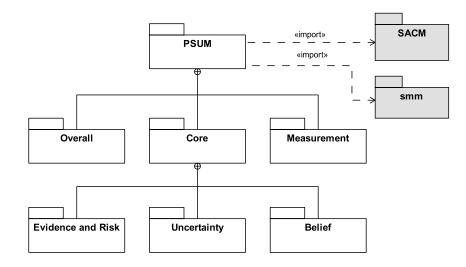


Figure 8.1: PSUM Package Structure

# 9 Overall

The *Overall* package defines the PSUM model and its elements (i.e., *PSUMElement*), as shown in the figure below. PSUM elements may have dependencies among them, which can be typed as *Derive*, *Trace*, *Use* and *Refine*. Each PSUM element may optionally be associated with annotations and may also have optional attributes (characterized by tag and value pairs). In addition to ID and description, a PSUM element may also be characterized by a point in time, indicating when the model element was specified.

Also, notice that an association is added in the PSUM metamodel to indicate that measurements of measurable PSUM elements (e.g., uncertainty) are specified with the SMM standard [10].

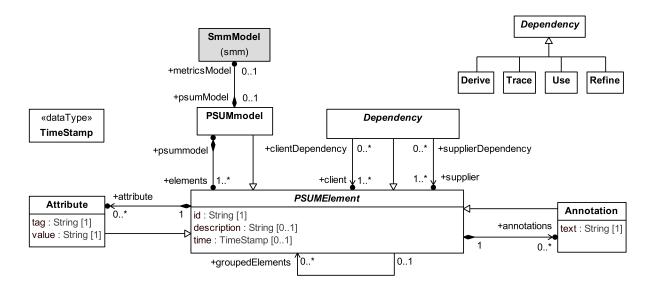


Figure 9.1: PSUM Overall Package

### 9.1 PSUMelement [Class]

*PSUMElement* is a constituent of a PSUM model. All the other model elements of the PSUM metamodel specialize *PSUMElement*, though these specializations are not shown in the figures. A *PSUMElement* can optionally have a *TimeStamp* representing a point in time.

### 9.2 Dependency [Class]

*Dependency* represents that a single model element, or a set of model elements, requires other model elements for their specification or implementation. This means that the complete semantics of the client element(s) are either semantically or structurally dependent on the definition of the supplier element(s). [11]

### 9.3 PSUMmodel [Class]

*PSUMmodel* consists of a set of PSUM elements and SMM model elements. The latter are subsumed in a *SmmModel* connected to a *PSUMmodel*.

### 9.4 Attribute [Class]

*Attribute* allows information to be attached to any model element in the form of a tagged value pair. An associated constraint does not allow *Attributes* to have *Attributes* or *Annotations* attached to them, as formally specified below:

context Attribute inv NoAttributeOrAnnotationOnAttribute:

self.attribute->size()=0 and self.annotations->size()=0

#### 9.5 Annotation [Class]

Annotation allows textual descriptions to be attached to any instance of a model element [10]. An associated constraint does not allow Annotations to have Attributes or Annotations attached to them, as formally defined below:

context Annotation inv NoAttributeOrAnnotationOnAnnotation: self.attribute->size()=0 and self.annotations->size()=0

### 9.6 Derive [Class]

*Derive* specifies a derivation relationship among model elements that are usually, but not necessarily, of the same type. Such a dependency specifies that the client may be computed from the supplier [11].

### 9.7 Trace [Class]

*Trace* specifies a trace relationship between model elements or sets of model elements that represent the same concept in different models [11].

### 9.8 Use [Class]

*Use* is a dependency that one *PSUMElement* requires another *PSUMElement* (or set of *PSUMElements*) for its full implementation or operation. The *Use* dependency does not specify how the client uses the supplier other than the fact that the supplier is used by the definition or implementation of the client [11].

### 9.9 Refine [Class]

*Refine* specifies a refinement relationship between model elements at different semantic levels, such as analysis and design [11].

# 10 Belief

The PSUM belief metamodel package captures *Beliefs* made by *BeliefAgents*. A belief is composed of at least one belief statement. A belief statement can be associated with a set of uncertainties. Both a belief and an uncertainty can be associated with indeterminacy sources. A belief is made based on *PersonalExperience* and/or *Evidence*. A belief itself can be a basis to support the proposal of another belief.

Between *BeliefStatement* and *Uncertainty*, *UncertaintyTopic* is captured, which is defined to relate the uncertainty to its subject (i.e., the associated *BeliefStatement* about which exists uncertainty). Furthermore, a *BeliefStatement* can include more than one associated *UncertaintyTopics*. An *UncertaintyTopic* can be associated with one or more *BeliefStatements*. For example, the "wind speed" *UncertaintyTopic* can be discussed in multiple *BeliefStatement* elements. Also, the same *UncertaintyTopic* can be discussed in more than one *BeliefStatement* described using different languages (e.g., graphical modeling languages such as UML, or natural-language statements in English).

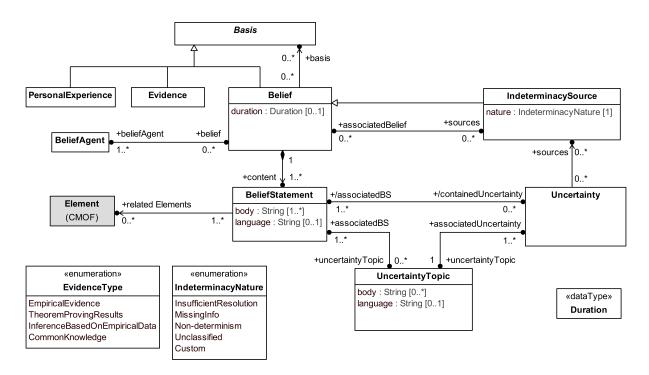


Figure 10.1: PSUM Belief Model

### 10.1 Belief [Class]

*Belief* is an implicit subjective explanation or conceptualization of some phenomena or notions, which is held by a *BeliefAgent*. Note that the term "phenomena" is intended to cover aspects of objective reality, whereas "notion" covers abstract concepts, such as those encountered in mathematics or philosophy [6].

In PSUM, implicit *Belief* is distinguished from *BeliefStatement*, which is an explicit representation of (parts of) a *Belief*. Beliefs are handled as explicitly specified content; therefore, a belief must have at least one *BeliefStatement*. The following two constraints are applied to *Belief*:

The indeterminacy sources associated to an uncertainty should be included in the set of indeterminacy sources of the corresponding belief:

context Belief inv ConsistentSources: self.sources->includesAll(self.content.containedUncertainty.sources)

A belief cannot form part of the basis for itself:

```
context Belief inv NotSelfBasis:
self.basis->closure(b|b.oclAsType(Belief).basis)->excludes(self)
```

A belief can optionally have its *duration* specified, which indicates the time during which the belief is considered valid.

### 10.2 BeliefAgent [Class]

*BeliefAgent* is a physical entity that holds (i.e., owns) one or more beliefs about phenomena or notions associated with one or more subject areas. This could be a human individual or group, an institution, a living organism, or even a machine such as a computer [6]. Crucially, a belief agent is capable of actions based on its beliefs [6].

### 10.3 BeliefStatement [Class]

*BeliefStatement* is a concrete, tangible representation of (parts of) a belief, specified according to the syntactic rules of a specific language. A *BeliefStatement* can have one or many bodies, which share the same set of *Uncertainty*. For example, the same content can be specified in different natural languages; the same content ("the door is open") can be specified as different belief statements in the same language ("the door is open" and "the door is 3 centimeters to the door frame.").

### 10.4 Uncertainty [Class]

*Uncertainty* is the state of deficiency (e.g., lack or partiality) of information or knowledge required to understand the content, consequence or likelihood of an *UncertaintyTopic* existent in a *BeliefStatement*.

### 10.5 UncertaintyTopic [Class]

UncertaintyTopic relates an Uncertainty to its subject (i.e., the associated BeliefStatement).

Role *containedUncertainty* of the association between *BeliefStatement* and *Uncertainty* is a derived role. Uncertainties contained in a *BeliefStatement* can be derived through *UncertaintyTopic*:

context BeliefStatement::containedUncertainty : Set(Uncertainty) derivedSet: self.uncertaintyTopic.associatedUncertainty->asSet()

### 10.6 IndeterminacySource [Class]

*IndeterminacySource* is a situation where the information required to ascertain the validity of a belief statement is indeterminate in some way, resulting in uncertainty being associated with that statement [6].

### 10.7 IndeterminacyNature [Enumeration]

IndeterminacyNature specifies different kinds of indeterminacy that characterize an IndeterminacySource. [6]

- *InsufficientResolution* denotes that the information available about the phenomenon in question is not sufficiently precise;
- *MissingInfo* denotes that the full set of information about the phenomenon in question is unavailable at the time when the statement is made;
- Non-determinism denotes that the phenomenon in question is either practically or inherently non-deterministic;
- Unclassified denotes indeterminate indeterminacy;
- *Custom* is a user-defined indeterminacy.

### 10.8 Basis [Class]

Basis is the basis on which a BeliefAgent makes a Belief.

### 10.9 PersonalExperience [Class]

*PersonalExperience* is also named as anecdotal evidence, which is defined as testimony that something is true, false, related, or unrelated based on isolated examples of someone's personal experience.

#### 10.10 Evidence [Class]

*Evidence* is either an observation or a record of a real-world event occurrence or, alternatively, the conclusion of some formalized chain of logical inference that provides information that can contribute to determining the validity (i.e., truthfulness) of a *Belief*. It is inherently an objective phenomenon representing something that actually happened, which implies that the possibility of counterfeit or invented evidence is excluded. Nevertheless, although *Evidence* represents objective reality, it need not be conclusive in the sense that it removes all doubts (*Uncertainty*) about a *BeliefStatement*.

#### 10.11 EvidenceType [Enumeration]

*EvidenceType* consists of four literals:

- *EmpiricalEvidence* representing information acquired by observation or experimentation;
- TheoremProvingResults denoting evidence collected from results of theorem proving;
- InferenceBasedOnEmpiricalData representing Information collected from inference based on empirical data;
- CommonKnowledge denoting knowledge known to everyone or nearly everyone.

# 11 Uncertainty

This part of the PSUM metamodel describes concepts that characterize uncertainty, via the abstract metaclass *UncertaintyCharacteristic* and its subtypes: *UncertaintyPerspective*, *Pattern*, and *Effect*. Uncertainty is further characterized by its attributes: kind, nature, and reducibility. Classifications of uncertainty based on these three attributes can be described using the three enumerations: *UncertaintyKind*, *UncertaintyNature*, and *ReducibilityLevel*.

In addition, this part of the metamodel captures the evolution of uncertainty. For this, the metamodel defines *UncertaintyHistory*. The history of an uncertainty captures a time series of changes made to its characteristics. Often, uncertainty measurements are changed over time. This is captured via the association from *UncertaintyHistory* to Measurement of SMM.

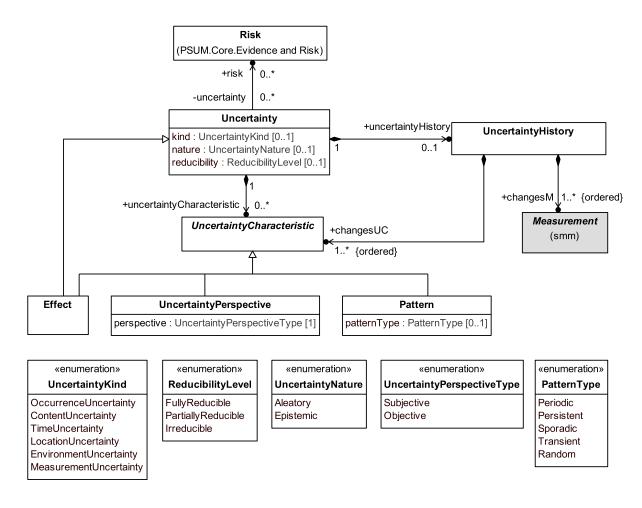


Figure 11.1: PSUM Uncertainty Model

### 11.1 Uncertainty [Class]

*Uncertainty* is the state of deficiency (e.g., lack or partiality) of information or knowledge required to understand the content, consequence or likelihood of an *UncertaintyTopic* existent in a *BeliefStatement*.

The following integrity constraints apply to class Uncertainty.

Aleatory uncertainty is irreducible, in that there will always be variability in the underlying variables.

```
context Uncertainty inv AleatoryIsIrreducible:
self.nature = UncertaintyNature::Aleatory
implies self.reducibility=ReducibilityLevel::Irreducible
```

Epistemic uncertainty is reducible, in that additional information or knowledge may reduce it.

```
context Uncertainty inv EpistemicIsReducible:
self.nature = UncertaintyNature::Epistemic implies
    (self.reducibility = ReducibilityLevel::PartiallyReducible or
    self.reducibility = ReducibilityLevel::FullyReducible)
```

Only occurrence uncertainties can have one pattern.

```
context Uncertainty inv NoPatternForNonOccurrenceUncertainty:
```

```
self.kind <> UncertaintyKind::OccurrenceUncertainty implies
```

self.uncertaintyCharacteristic->select(oclIsKindOf(Pattern))->size()=0

One may or may not define a pattern for an occurrence type of uncertainty.

```
context Uncertainty inv PatternForOccurrenceUncertainty:
self.kind = UncertaintyKind::OccurrenceUncertainty implies
self.uncertaintyCharacteristic->
    select(oclIsKindOf(Pattern))->size()<=1</pre>
```

#### 11.2 UncertaintyCharacteristics [Class]

UncertaintyCharacteristics is about information that in some way characterizes the associated uncertainty. It is specialized into the more detailed characteristics *Effect*, *UncertaintyPerspective*, and *Pattern*.

### 11.3 UncertaintyHistory [Class]

UncertaintyHistory keeps the record of a sequence of changes applied to any UncertaintyCharacteristic or Measurement of an Uncertainty.

#### 11.4 UncertaintyPerspective [Class]

UncertaintyPerspective is an uncertainty characteristic that specifies whether an uncertainty is subjective or objective in terms of the perspective of observing agents.

#### 11.5 UncertaintyPerspectiveType [Enumeration]

UncertaintyPerspectiveType defines the following types:

- *Subjective* uncertainty perspective refers to information existing within some agency derived from observation and/or reasoning by that agency;
- *Objective* uncertainty perspective refers to phenomena or concepts whose existence and nature are independent of any observing agency.

If an uncertainty is fully reducible, then it is possible to transit from subjective to objective when having enough evidence.

### 11.6 Pattern [Class]

Pattern is an intelligible way in which an uncertainty appears.

### 11.7 PatternType [Enumeration]

PatternType defines the following types - from [6]:

- Periodic pattern occurs at regular intervals of time;
- Persistent pattern lasts forever;
- Sporadic pattern occurs occasionally;
- *Transient* pattern occurs temporarily;
- Random pattern occurs without a definite method, purpose or conscious decision.

#### 11.8 Effect [Class]

*Effect* captures the result of an uncertainty in a belief statement, i.e., the consequence of misinterpreting the *BeliefS*-*tatement* due to uncertainty associated with it. An uncertainty may result in another known uncertainty; therefore, an *Effect* itself can be another *Uncertainty*.

#### 11.9 UncertaintyKind [Enumeration]

UncertaintyKind includes the following types - from [6]:

- *ContentUncertainty* represents a situation whereby a *BeliefAgent* is uncertain about the content expressed in a *BeliefStatement*;
- *EnvironmentUncertainty* represents a situation whereby a *BeliefAgent* is uncertain about the surroundings of a physical system existing in a *BeliefStatement*;
- *GeographicalLocationUncertainty* represents a situation whereby a *BeliefAgent* is uncertain about the geographical location existing in a *BeliefStatement*;
- *OccurrenceUncertainty* represents a situation whereby a *BeliefAgent* is uncertain about the occurrence of events existing in a *BeliefStatement*;
- *TimeUncertainty* represents a situation whereby a *BeliefAgent* is uncertain about time existing in a *BeliefStatement*.

#### 11.10 ReducibilityLevel [Enumeration]

ReducibilityLevel includes the following types:

- *FullyReducible* uncertainty denotes that there is no full certainty, but uncertainty is knowable and can be reduced by collecting additional information until achieving full certainty (no irreducible uncertainty present);
- *PartiallyReducible* uncertainty means that there is no full certainty, but uncertainty can be reduced by collecting additional information;
- *Irreducible* uncertainty means that there is no full certainty, and it cannot be reduced.

#### 11.11 UncertaintyNature [Enumeration]

UncertaintyNature of an uncertainty indicates whether it is of the aleatoric nature or epistemic nature.

- Aleatory uncertainty subsumes uncertainty caused by the inherent stochasticity of real-world phenomena;
- Epistemic uncertainty is caused by lack of information [12].

# 12 Measurement

The measurement part of the PSUM metamodel aims to provide a generic approach to quantify *Belief*, *Uncertainty*, *Risk*, and *IndeterminacySource*, which are all considered as *MeasurableElements* of the PSUM metamodel. A measurable element may be composed of a list of measurable features, which can be accuracy, sensitivity, measurement error, precision, and degree. In addition, a measurable feature is associated with *Measurement* of the SMM specification, through which a measurable element (e.g., the degree of a belief) can be measured, with a specific Measure (defined in the SMM standard [10]).

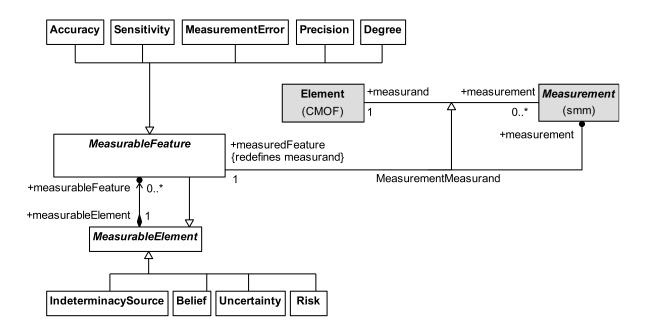


Figure 12.1: PSUM Measurement Package

### 12.1 MeasurableFeature [Class]

*MeasurableFeature* of a *MeasurableElement* quantifies it by a *Measurement*, a numerical or symbolic value assigned to the *MeasurableElement* by a *Measure* [10].

### 12.2 MeasurableElement [Class]

*MeasurableElement* is a *PSUMElement* that can be measured. A *MeasurableElement* is decomposed by a set of associated *MeasurableFeature* instances, i.e., there might be multiple different aspects that can be measured in order to quantify a *MeasurableElement*.

### 12.3 Risk [Class]

Risk is the effect of uncertainty on objectives [1].

#### 12.4 Accuracy [Class]

Accuracy measures the closeness of agreement between a quantity value obtained by measurement and the true value of the measurand [13].

#### 12.5 Sensitivit [Class]

*Sensitivity* is defined as the quotient of the change in an indication of a measuring system and the corresponding change in a value of a quantity being measured [13]. Sensitivity is an absolute quantity, the smallest absolute amount of change that can be detected by a measurement.

#### 12.6 MeasurementError [Class]

*MeasurementError* is defined as the result of a measurement minus a true value of the *measurand* [13]. Also called "absolute error of measurement", in order to distinguish it from "relative error" which is the measurement error divided by a true value of the measurand [13].

#### 12.7 Precision [Class]

*Precision* measures the closeness of agreement between quantity values obtained by replicate measurements of a quantity, under specified conditions. Measurement precision is usually expressed numerically by measures of imprecision, such as standard deviation, variance, or the coefficient of variation under the specified conditions of measurement [13].

#### 12.8 Degree [Class]

*Degree* is one kind of *MeasurableFeature* that can be used to measure *IndeterminacySource*, *Belief*, *Risk* and *Uncertainty*, by indicating their extent, according to either a scale, a numerical value such as a probability or any other measure of subjective uncertainty (e.g., a fuzzy value, a subjective logic opinion).

- Degree of IndeterminacySource captures the extent to which there is uncertainty about the IndeterminacySource.
- Degree of Belief captures the extent to which a belief agent is confident about the validity of the Belief.
- Degree of *Risk* captures the severity of a *Risk* associated with uncertainty.
- Degree of Uncertainty captures the extent to which the Uncertainty persists.

# 13 Evidence

The PSUM metamodel only defines *Evidence*, as the SACM specification [9] does not define this concept explicitly. Through *Evidence*, the PSUM metamodel is connected to SACM, specifically its concepts of *ArtifactElement* and *ArtifactElement* citation. In SACM, *ArtifactElement* is defined as "objective artifacts being offered in support of one or more claims" [9]. In the PSUM metamodel, the enumeration *EvidenceType* is defined, which provides an opportunity to characterize evidence at a very high level. Users are encouraged to associate PSUM to SACM for comprehensive modeling of evidence when needed.

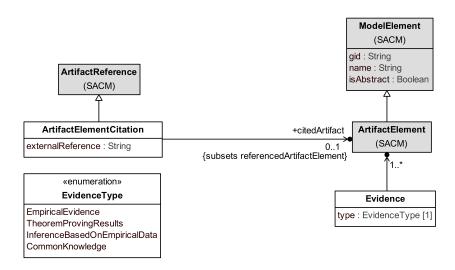


Figure 13.1: PSUM Evidence Model

#### 13.1 Evidence [Class]

*Evidence* (see its definition in Section 10) is associated with a set of *ArtifactElement* of SACM, which is defined as "objective artifacts being offered in support of one or mode claims". [9]. See also the definition of *EvidenceType* in Section 10.

#### 13.2 ArtifactElementCitation [Class]

Class ArtifactElementCitation specializes SACM Class ArtifactReference, identifying a specific artifact referenced as evidence.

# Appendix A Examples of Applying PSUM

(informative)

## A.1 Specifying Uncertainty Requirements

This case study is a partial representation of beliefs and uncertainties specified in the use case specification (UCS) presented in [14]. In [14], a methodology, named U-RUCM, was presented to specify uncertainty requirements as part of use case models, which is an integration of the Restricted Use Case Modeling (RUCM) [15] and U-Model [6]. A smart home system implements various safety and security features such as detecting intrusions and fires. One key use case is about a homeowner enabling windows and doors monitoring of her/his home. The UCS of this use case has the basic flow describing the happy path that the monitoring is enabled properly, and the system is running as expected, and there is no intrusion detected. The UCS also contains several alternative flows such that any occurrence of an intrusion should be detected, immediately followed by sending an intrusion notification to the homeowner and the activation of an alarm. PSUM specifies uncertainty requirements on this UCS, as described below.

The object diagram below defines the *BeliefAgent SRL* (a requirements engineer, a domain expert, or even a smart home development team), who makes all the beliefs presented in the diagram. *SRL* makes a belief on the overall UCS, i.e., *overall UCS belief : Belief*, which has a timestamp ("2021-03-11"), an associated *BeliefStatement* (i.e., a *brief description of the UCS*: "The system monitors the status of windows and doors when Home Owner enables the monitoring function", in which Home Owner is the primary actor the use case), an associated *Evidence* instance, an associated *IndeterminacySource* instance (i.e., *broken control panel*). In addition, the belief degree is measured via a *MeasurableFeature* instance and an instance of *DirectMeasurement* from SMM, i.e., 80% confidence (in probability) on the belief. The belief statement corresponds to the brief description of the UCS, which summarizes the goal of the UCS, which has its body specified as an English statement: "The system monitors the status of windows and doors when Home Owner enables the monitoring function."

In addition, in the object diagram shown below specifies beliefs and belief statements corresponding to the precondition of the UCS (as shown in *precondition of UCS : BeliefStatement*) and one of the steps of the UCS's basic flow (i.e., step 5). The *step 5* belief statement (*belief on step 5 of the UCS's basic flow : Belief*) has its body as "The system enables the monitoring function.", which is also an English statement. The belief statement is associated with an *Uncertainty* (i.e., does *not enable*) via the *enable : UncertaintyTopic*. The uncertainty is characterized by its *Effect*, *UncertaintyPerspective*, *Pattern*, and uncertainty measurement. In addition, the model kept the history of the change made to the uncertainty measurement: from 10% probability to 2% probability.

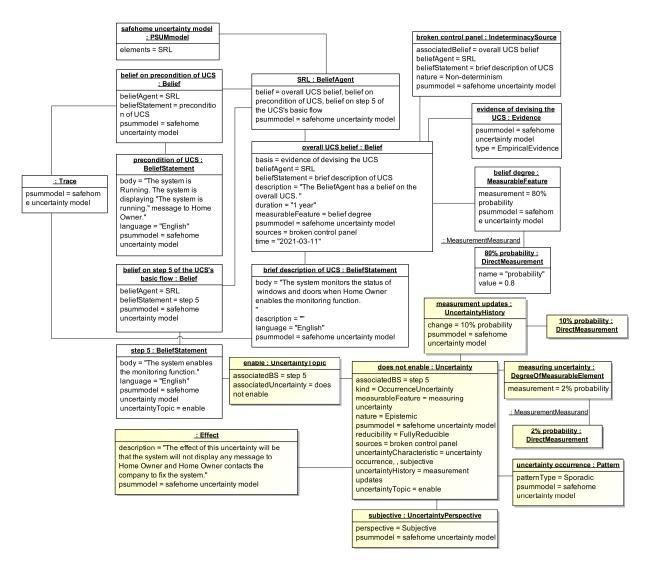


Figure A.1: PSUM Safe Home Example

## A.2 Measurement Uncertainties

The following models show how the PSUM metamodel can be used to specify different kinds of measurements and belief uncertainties.

## A.2.1 Indicating the Precision of a Measurement

The first example shows the specification of the statement:

"Betty thinks that the length of the room is 3m with a precision of 0.001m."

In this case, the *Belief* is represented by one *BeliefStatement*, which is associated to one *UncertaintyTopic* ("the length of the room"), and to a given *Uncertainty*, of kind *MeasurementUncertainty* and hence of the *Aleatory* nature. In this case, the measurable feature of interest of the *Uncertainty* is *Precision*, and the model element related to the *BeliefStatement* is the length of the room, another measurable feature. To measure these two measurable features, the

SMM is used. The corresponding SMM elements are shown with unshaded boxes. They assign the concrete values to the measurable features by specifying how they were measured and their units.

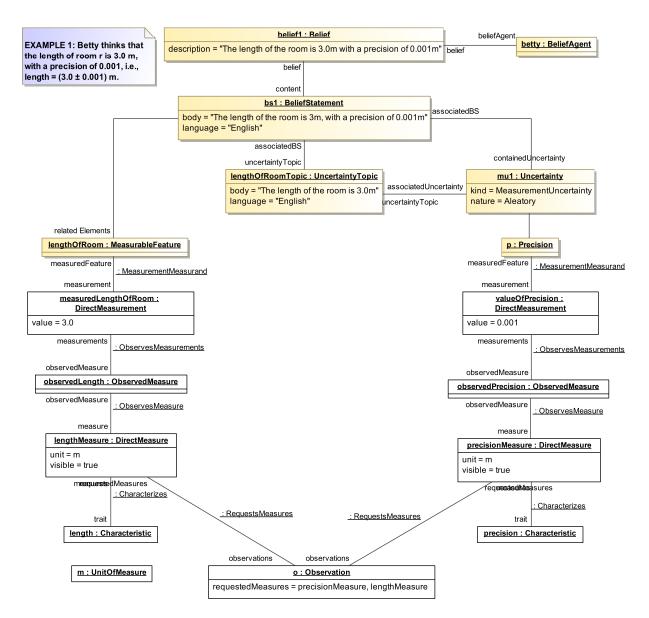


Figure A.2: Precision of Measurement Example

## A.2.2 Adding Confidence to a Belief

A second example shows how to add a degree of belief to the previous example. The model below shows the specification of the statement:

"Betty thinks, with a confidence of 0.95, that the length of the room is 3m with a precision of 0.001m."

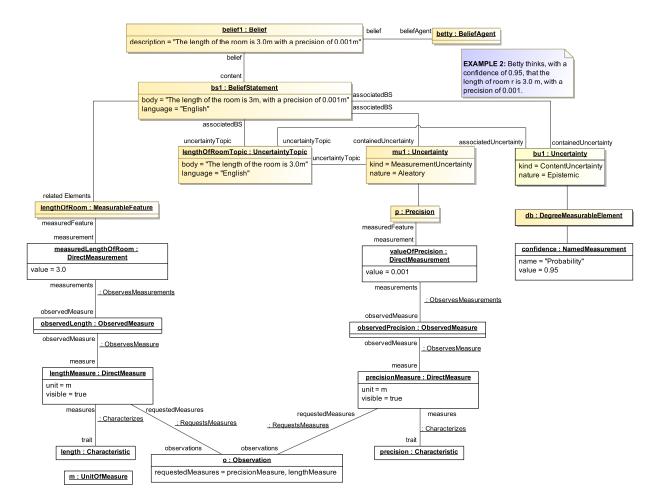


Figure A.3: Adding Confidence to Belief Example

Again, the unshaded boxes represent the SMM instances that specify the value of the measurements, how they are measured, and their units.

### A.2.3 Working with two or more Belief Statements

The third example shows how to add two measurement uncertainties to a value. The model below shows the specification of the statement:

"Betty thinks that the length of the room is 3m with a precision of 0.001m and an accuracy of 0.002m."

Here the Belief is expressed using two separate BeliefStatements.

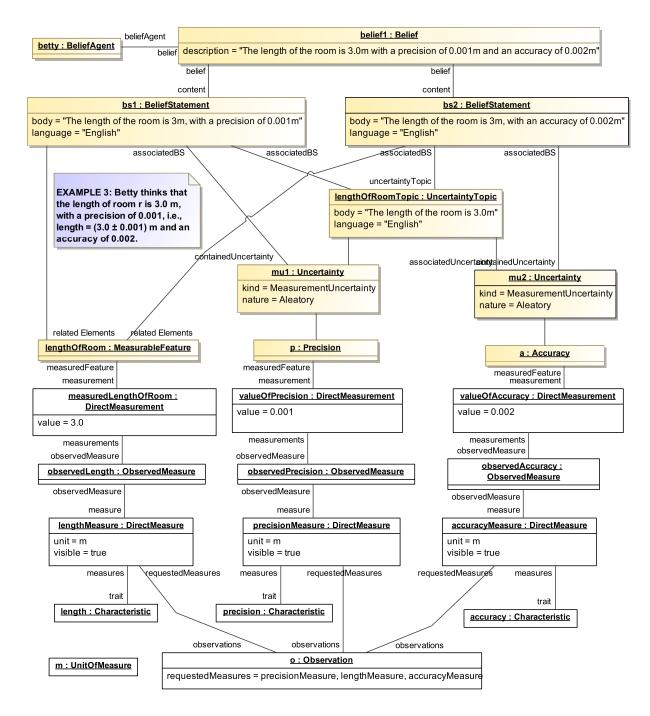


Figure A.4: Working with multiple Belief Statements Example

## A.2.4 Assigning Uncertainty to Uncertainty

This example shows how to add uncertainty to a level of uncertainty already specified. The model below shows the specification of the statement:

"Betty thinks that the length of the room is 3.0m with a precision of 0.001m, and that the accuracy of the measurement of the precision is 0.002m."

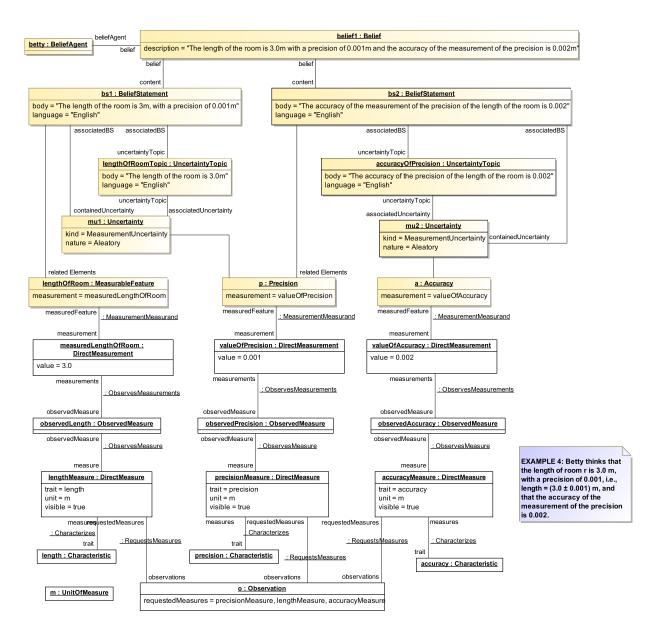


Figure A.5: Assigning Uncertainty to Uncertainty Example

Notice the difference with the previous example, where both precision and accuracy were associated to the same measurement (the length of the table is 3m), while associating the accuracy to the measurement of the precision.

## A.3 Operational Uncertainty due to Inconsistent Information

Our third example takes an operational perspective on uncertainty faced from the point of view of a software-intensive system during operation [16]. During the development of software-intensive systems, it is important to consider and understand the characteristics of the environment (or context) in which the system under development is supposed to operate. To that end, the operational context is analyzed and modeled. Especially in early development phases, e.g., requirements engineering, it is important to identify and understand the uncertainty that may occur when the system is in operation so that it can be equipped with capabilities to remain functional.

One situation that may occur especially in embedded and cyber-physical systems (e.g., in the automotive domain) is inconsistent information provided by different sensors monitoring the context. Autonomous vehicles process a broad range of context information. Such context information is acquired either through sensors or via communication between collaborating vehicles. The following object diagram shows uncertainty faced by some controller software systems embedded in a car, which needs to consolidate information obtained from different sources.

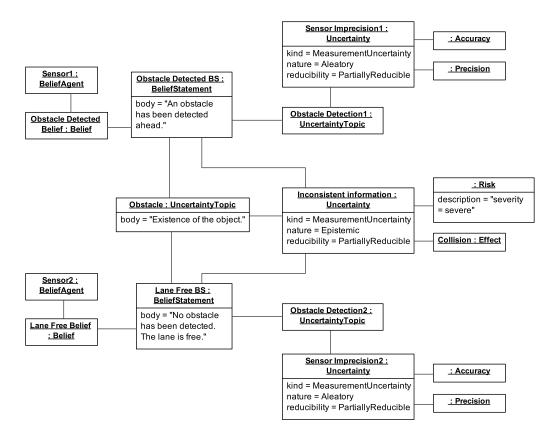


Figure A.6: Operational Uncertainty Example

In this example, there are two redundant onboard sensors installed in a vehicle (e.g. a Radar and a Lidar sensor). Both sensors monitor the road and are utilized to detect obstacles (such as other cars) ahead of the vehicle under consideration. These sensors are thus modeled as *BeliefAgent* instances. Given the measurements obtained from these sensors, *Belief* instances characterizing the current state of the vehicle's surroundings can be derived. Consider the situation of inconsistent *Belief* instances about the road ahead: The measurement obtained from *Sensor1* indicates that there is an obstacle (*Obstacle Detected Belief*) while the other sensor's (*Sensor2*) measurements do not (modeled as the *Lane Free Belief* instance). Using the *Belief* part of PSUM, respective representations of these two beliefs were captured as instances of the *BeliefStatement* concept. Note that these pieces of information are considered on a conceptual level, and without considering their concrete form, i.e., in a language-independent way.

Both *BeliefStatements* on their own are subject to uncertainty due to the inherent imprecision of the two sensors. The two belief statements were connected to respective *Uncertainty* instances using the two *UncertaintyTopic* instances *Obstacle Detection1* and *Obstacle Detection2*. These uncertainties are both categorized as *MeasurementUncertainty* and can be quantified when considered as *MeasurableElements* using the *Measurement* part of PSUM. That way, it is possible to associate, e.g., *Precision* and *Accuracy* measurements. Since the previous examples have already elaborated in detail on capturing measurement uncertainty (see Annex A.2), we do not go into more details here. The two *Uncertainty* instances *Sensor Imprecision1* and *Sensor Imprecision2* are characterized as aleatory, as they are caused by physical aspects of sensor technology and the context. Furthermore, they are considered partially reducible, because, in principle, obtaining more information, such as through other, redundant sensors, helps reduce the uncertainty.

Nevertheless, merging information from two different sources also introduces another *Uncertainty* instance. From the point of view of the controller software during operation, which processes the two pieces of information in order to react by e.g. braking, this additional uncertainty relates to both *BeliefStatements* in combination. This is captured in the model through the additional *Uncertainty* instance *Inconsistent Information*. This uncertainty is connected to the two *BeliefStatements* mentioned above using *UncertaintyTopic* instances, and explicitly refers to both these statements in combination. The uncertainty topic can thus be phrased as "Existence of object". The associated uncertainty is categorized as *Epistemic* because it can be considered on a higher, less technical level compared to the other two measurement uncertainties. The focus is on comparing two contradicting pieces of information rather than on the originating sources of this information. However, this uncertainty is also only partially reducible since the measurement uncertainty related to the original *Belief* instances remains.

The model also contains elements characterizing the *Risk* and *Effect* of the uncertainty. This is especially important when developing safety-critical systems such as embedded software in a vehicle, as in our example. There is a severe *Risk* instance that a *Collision* with an obstacle may occur as an *Effect*, in case the obstacle really exists. Thus, PSUM helps conceptualize and analyze uncertainty that may potentially occur during operation. From a software engineering perspective, domain-specific languages can be designed based on PSUM to support engineers in systematically building countermeasures into the system so that it will be able to cope with the defined uncertainty autonomously during operation.

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